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(Residential Autonomous College affiliated to University of Calcutta)
B.A./B.Sc. FOURTH SEMESTER EXAMINATION, MAY 2018
SECOND YEAR (BATCH 2016-19)
Date : 25/05/2018 **MATH FOR ECONOMICS (General)**
Time : 11.00 am – 2.00 pm Paper : IV Full Marks : 75

[Use a separate Answer Book for each Group]

Group-A

Answer any five questions [5×5]

1. a) Let $T: P_2(\mathbb{R}) \to P_3(\mathbb{R})$ be the linear transformation defined by

$$T(f(x)) = 2f'(x) + 3\int_0^x 3f(t)\ dt$$

Determine $[T]_\beta^\gamma$ where β and γ are the standard ordered bases of $P_2(\mathbb{R})$ and $P_3(\mathbb{R})$. 3

b) Define the geometric and algebraic multiplicity of an eigenvalue of a linear operator *T*. 2

2. a) Use Cayley-Hamilton theorem to find A^{50}, where $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. 3

b) Let, characteristic polynomial for the matrix *A* is $f(x) = (x-1)^2(x-3)$. Find $\det A^{-1}$. 2

3. a) Prove that two similar matrices have the same eigenvalues. 3

b) "Every finite dimensional linear operator have eigenvalues and eigenvectors" is it true? Explain. 2

4. a) State the necessary and sufficient condition for the diagonlisibility of a square matrix. 2

b) Let the characteristic polynomial of a matrix *A* be $f(x) = (x-1)(x-3)(x-4)$, then whether *A* is diagonalizable and if *A* is diagonalizable then what is the diagonal matrix corresponding to A? 3

5. Diagonalize the symmetric matrix $A = \begin{bmatrix} 1 & 3 & 5 \\ 3 & 1 & -2 \\ 5 & -2 & 2 \end{bmatrix}$. 5

6. a) Find λ for which the quadratic form $f(x, y, z)$ is positive definite, where

$f(x, y, z) = x^2 + \lambda(y^2 + z^2) + 2yz$. 3

b) Show that the real quadratic form $ax^2 + 2hxy + by^2$ $(a \neq 0)$ in two variables *x*, *y* is negative definite if and only if $a < 0$ and $\begin{vmatrix} a & h \\ h & b \end{vmatrix} > 0$. 2

7. In $\mathbb{P}_2(\mathbb{R})$, define two inner products by,

$$\langle f, g \rangle_1 = \int_{-1}^{1} f(x)\ g(x)\ dx$$

$$\langle f, g \rangle_2 = \int_0^1 f(x)\ g(x)\ dx$$

Then give an example of two elements $f, g \in \mathbb{P}_2(\mathbb{R})$ such that $\langle f, g \rangle_1 = 0$ but $\langle f, g \rangle_2 \neq 0$. Also state what can you conclude from this observation? 5

8. a) Prove that convex polyhedron is a convex set. 3

b) If x_1, x_2 be real, show that the set given by $X = \left\{(x_1, x_2) \middle| 9x_1^2 + 4x_2^2 \le 36\right\}$ is a convex set. 2

Group-B

Answer <u>any six</u> questions [6×5]

9. a) Define feasible solution and basic feasible solution of an LPP. 2

b) Prove that the set of all feasible solution of an linear programming is a convex set. 3

10. a) State the fundamental theorem of linear programming. 2·5

b) State the fundamental duality theorem. 2·5

11. a) Solve the following LPP graphically. 3

$$\begin{aligned} \text{Maximize} \quad & z = 3x_1 + x_2 \\ \text{Subject to} \quad & 2x_1 + 3x_2 \ge 18 \\ & x_1 + 2x_2 \le 6 \\ & x_1,\ x_2 \ge 0 \end{aligned}$$

b) Show that the following system of linear equations has two degenerate basic feasible solution and the non-degenerate basic solution is not feasible. 2

$$3x_1 + x_2 - x_3 = 3$$
$$2x_1 + x_2 + x_3 = 2$$

12. Food X contains 5 units of vitamin A and 6 units of vitamin B per gram and costs 20 p/gm. Food Y contains 8 units of vitamin A and 10 units of vitamin B per gram and costs 30 p/gm. The daily requirement of A and B are atleast 80 and 100 units respectively. Formulate the above as a linear programming problem to minimize the cost. 5

13. Consider the LPP 5

$$\begin{aligned} \text{Minimize} \quad & z = x_1 + 3x_2 + 2x_3 \\ \text{Subject to} \quad & x_1 - 2x_2 + x_3 = 2 \\ & 2x_1 - x_2 \le 5 \\ & x_2 - 2x_3 \ge 3 \end{aligned}$$

Formulate the dual of the LPP and also verify that dual of a dual is primal in this case.

14. Use Charne's Big-M method to solve the LPP [5]

$$\begin{aligned} \text{Maximize} \quad & z = x_1 + 5x_2 \\ \text{Subject to} \quad & 3x_1 + 4x_2 \le 6 \\ & x_1 + 3x_2 \ge 3 \end{aligned}$$

where $x_1, x_2 \ge 0$

15. Use two phase simplex method to solve the following LPP. 5

Maximize $z = 2x_1 + x_2 + x_3$

Subject to $4x_1 + 6x_2 + x_3 \le 8$

$3x_1 - 6x_2 - 4x_3 \le 1$

$2x_1 + 3x_2 - 5x_3 \ge 4$

$x_1,\ x_2,\ x_3 \ge 0$

16. a) State the fundamental duality theorem. 2

b) Solve graphically. 3

Maximize $z = x + 3y$

Subject to $0 \le x \le 2$

$0 \le y \le 3$

$x + y \le 5$

$x, y \ge 0$

17. $x_1 = 2,\ x_2 = 3$ and $x_3 = 1$ is a feasible solution of the LPP. 5

Maximize $z = x_1 + 2x_2 + 4x_3$

Subject to $2x_1 + x_2 + 4x_3 = 11$

$3x_1 + x_2 + 5x_3 = 14$

$x_1, x_2, x_3 \ge 0$

Group-C

Answer <u>any four</u> questions [4×5]

18. Find out the Nash Equilibrium for the following zero sum game: 5

		Column			
		North	South	East	West
Row	Up	6	7	5	6
	High	7	3	4	5
	Low	8	6	3	2
	Down	3	3	4	5

Describe the steps you have used to find the equilibria.

19. Consider the following table: 1+2+2

		Column			
		North	South	East	West
Row	Earth	1, 3	3, 1	0, 2	1, 1
	Water	1, 2	1, 2	2, 3	1, 1
	Wind	3, 2	2, 1	1, 3	0, 3
	Fire	2, 0	3, 0	1, 1	2, 2

(i) Does either row or column has any dominant strategy?

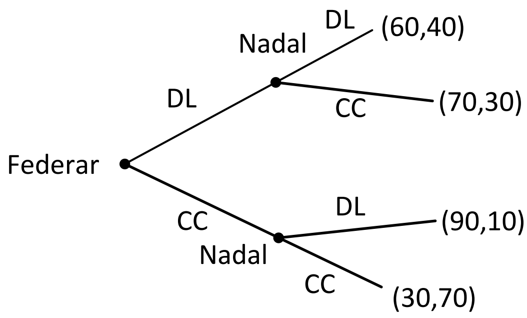
(ii) Use iterated elimination of dominated strategies to solve the game as much as possible.

(iii) Find the Nash Equilibrium of the game.

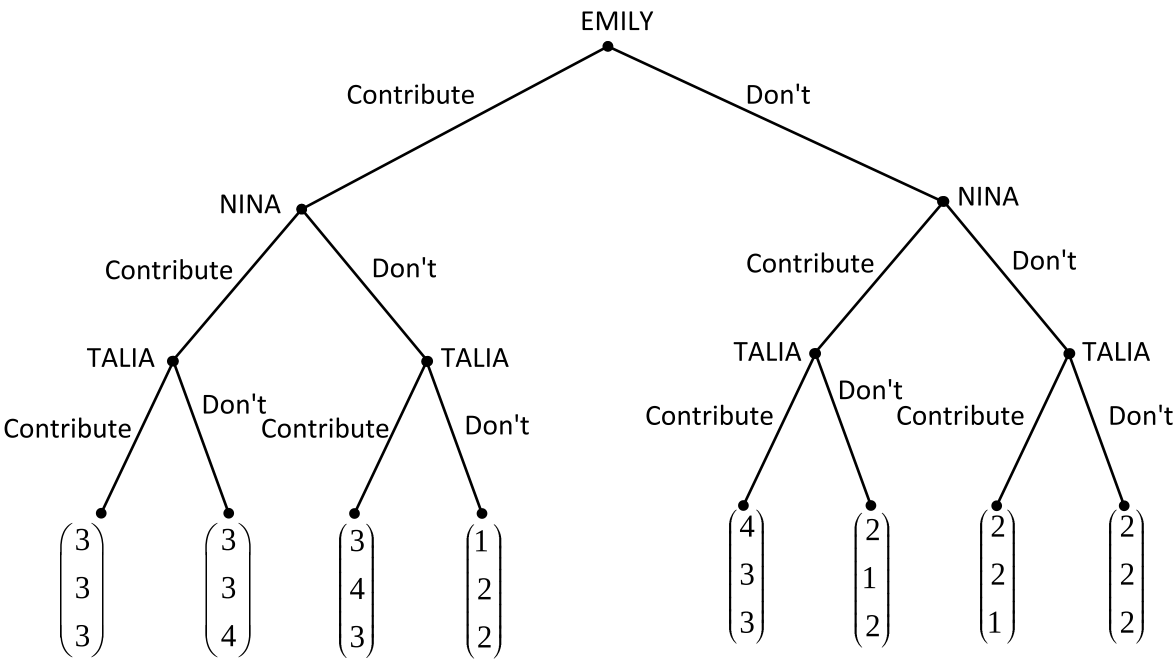
20. Find the value of the game and the optimal mixed strategy of each player. [5]

		Player B	
		B_1	B_2
Player A	A_1	2	5
	A_1	3	4

21. Find the Nash Equilibrium of the following game: [5]



22. Find the subgames of the following game: 5



23. Show that if in the *n*-player normal form game the Nash Equilibrium survive iterated elimination of strictly dominated strategies. 5

24. Find out the subgame perfect equilibrium of the following game: 5